

Quantum Mechanics

PROBLEM SET 1

due on Monday, January 14, 2008

1. These problems are intended as a short review of complex numbers.

(a) If $\psi = |\psi| \exp(i\alpha_1)$ and $\chi = |\chi| \exp(i\alpha_2)$, show that

$$|\psi + \chi|^2 = |\psi|^2 + |\chi|^2 + 2|\psi\chi| \cos(\alpha_1 - \alpha_2)$$

(b) Show that (i) $2\operatorname{Re} z = z + z^*$, (ii) $(\exp z)^* = \exp z^*$, (iii) $|\exp z| = \exp(\operatorname{Re} z)$

2. Use the expression $e^{i\theta} = \cos \theta + i \sin \theta$ to derive the following identities by comparing real and imaginary parts of some complex relations:

(a) $\cos(\theta_1 + \theta_2) = \cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2$

(b) $2 \sin \theta_1 \cos \theta_2 = \sin(\theta_1 - \theta_2) + \sin(\theta_1 + \theta_2)$

3. For each of the following complex numbers z , express the real part $\operatorname{Re} z$, the imaginary part $\operatorname{Im} z$, the complex conjugate z^* and the square of the modulus $|z|^2$ in terms of (given) real numbers a, b, c :

(a) $z = e^{ib}$

(b) $z = e^{a+ib}$

(c) $z = \frac{e^{ia}}{b + ic}$

(d) $z = (ae^{ib} + ic)^2$

Your homework solutions will be collected at the beginning of Monday's class. Then a 15-minute quiz will be given based on one of these problems.