Model Solution: Homework 9

Physics U603 Summer 1 2006

Hecht 8.1 A complete description of the state of polarization should state whether it is linear, elliptical, or circular, whether it is left- or right-handed (if circular or elliptical), and its orientation (if linear or elliptical). One approach is to express the electric field in the form $\vec{E} = \hat{\imath} E_{0x} \cos(kz - \omega t + \delta_x) + \hat{\jmath} E_{0y} \cos(kz - \omega t + \delta_y)$, and evaluate the polarization state based on the phase difference $\delta = \delta_y - \delta_x$ and the relative magnitude E_{0y}/E_{0x} of the two field components. While this is systematic, it does not offer as much insight as sketching out the behavior of electric field vector as a function of time at a fixed location ($z = 0$ is often convenient). I recommend using at least two methods, and working to reconcile any apparent discrepancies, to establish confidence.

(a) Factoring out the common factor, $\vec{E} = (\hat{\imath} - \hat{\jmath}) E_0 \cos(kz - \omega t)$. This is linearly polarized light with the electric field oscillating (with amplitude $\sqrt{2} E_0$) along the direction $(\hat{\imath} - \hat{\jmath}) / \sqrt{2}$, 45° from the positive $x-$axis and the negative $y-$axis.

An equivalent expression is $\vec{E} = \hat{\imath} E_0 \cos(kz - \omega t) + \hat{\jmath} E_0 \cos(kz - \omega t + \pi)$. The π phase difference corresponds to linearly polarized light and the polarization direction follows from determining $\vec{E}(z = 0, t = 0) = (\hat{\imath} - \hat{\jmath}) E_0$.

(b) Again, the linear polarization is evident after removing the common factor in $\vec{E} = (\hat{\imath} - \hat{\jmath}) E_0 \sin[2\pi(z/\lambda - \nu t)]$. This is exactly the same polarization state as in (a), linearly polarized at $-45°$ to the $x-$axis.

The more systematic approach (which may be overkill here) reformulates the electric field as $\vec{E} = \hat{\imath} E_0 \cos(kz - \omega t - \pi/2) + \hat{\jmath} E_0 \cos(kz - \omega t + \pi/2)$. Again, linear polarization follows from the π phase difference and the polarization direction can be evaluated at almost any time and direction (except $z = 0, t = 0$, where $\vec{E} = 0$).

(c) The $\pi/4$ phase difference (or any phase difference other than a multiple of $\pi/2$) is characteristic of elliptical polarization. At $z = 0$, the electric field $-\hat{\jmath} E_o/\sqrt{2}$ points along $-y$ at $t = 0$. As t increases, the $x-$component increase from zero, and the $y-$ component initially decreases, so the vector rotates counterclockwise in time, corresponding to left-handed rotation. The same conclusion follows after reformulating as $\vec{E} = \hat{\imath} E_0 \cos(kz - \omega t + \pi/2) + \hat{\jmath} E_0 \cos(kz - \omega t + 3\pi/8)$.

Establishing the orientation requires a little more work. The orientation of the ellipse is the direction in which the squared magnitude $E^2 = E_0^2 \left[\sin^2\phi + \sin^2(\phi - \pi/4)\right]$ of the electric field obtains its maximum value (abbreviating the varying part of the phase as $\phi = \omega t - kz$). This takes place where the derivative $dE^2/d\phi = 2E_0^2 \left[\sin\phi\cos\phi + \sin(\phi - \pi/4)\cos(\phi - \pi/4)\right]$ vanishes. Since $2\sin\theta\cos\theta = \sin 2\theta$, this leads to $\sin 2\phi + \sin(2\phi - \pi/2) = \sin 2\phi - \cos 2\phi = 0$, or $\tan 2\phi = 1$. Evaluation of $\vec{E}$ at the two resulting values $\phi = \pi/8$ and $\phi = 5\pi/8$ yields $\vec{E}(\phi = \pi/8) = (\hat{\imath} - \hat{\jmath}) E_0 \sin(\pi/8)$ and $\vec{E}(\phi = 5\pi/8) = (\hat{\imath} + \hat{\jmath}) E_0 \sin(5\pi/8)$. Since the second of these is larger in magnitude, it defines the major axis of the ellipse, which is thus located at $+45°$ from the $+x$−axis.

(d) As time increases, the phase $\omega t - kz$ advances from 0, where $\vec{E} = \hat{\imath} E_0$, to $\pi/2$, where $\vec{E} = -\hat{\jmath} E_0$, to π, where $\vec{E} = -\hat{\imath} E_0$, to $3\pi/2$, where $\vec{E} = \hat{\jmath} E_0$. Since $\vec{E}$ rotates clockwise as a function of time with a constant magnitude E_0, the polarization is right handed circular.

In the standardised form, $\vec{E} = \hat{\imath} E_0 \cos(kz - \omega t) + \hat{\jmath} E_0 \cos(kz - \omega t - \pi/2)$ (using $\cos(-x) = \cos x$). The phase difference $\delta = -\pi/2$ corresponds to a right-handed polarization and $E_{0x} = E_{0y}$ ensures that it is circular rather than elliptical.

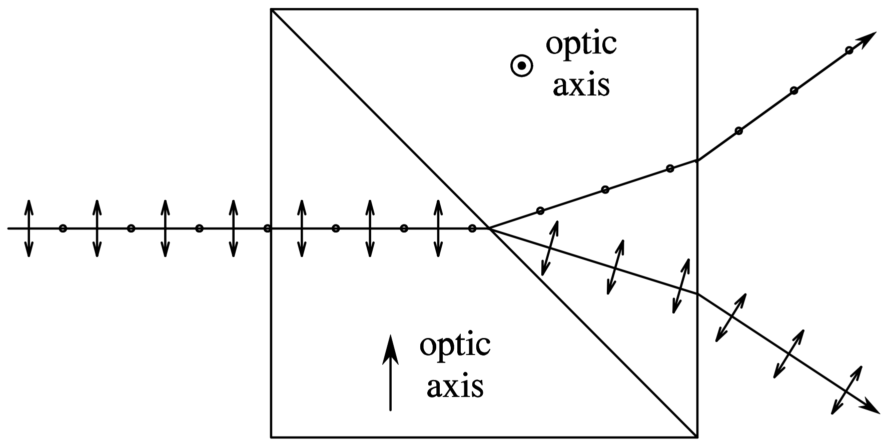
Hecht 8.12 "Natural" light has random polarization, so exactly half of the incident intensity makes it through the first polarizer. The emerging light is polarized parallel to the transmission axis of the first polarizer, or 40° from the transmission axis of the second. This reduces the net transmitted intensity to $I = \frac{1}{2} I_0 \cos^2\theta = 200\ \mathrm{W/m^2} \cos^2 40° = 117\ \mathrm{W/m^2}$.

Hecht 8.31 Since the reflected light is completely linearly polarized, the light must be incident at Brewster's angle $\theta_B = \tan^{-1}(n_2/n_1) = \tan^{-1}(1.65/1.36) = 50.5°$. Snell's Law gives the angle of the transmitted beam: $\sin\theta_2 = (n_1/n_2)\sin\theta_1$ or $\theta_2 = 39.5°$. Alternatively, the fact that the reflected and transmitted beams are orthogonal at Brewster's angle yields $\theta_2 = 90° - \theta_1 = 39.5°$.

Hecht 8.35 Table 8.1 on pg. 343 lists indices of refraction for several common materials. For the ordinary beam, $n_o = n_\perp = 1.5443$ for quartz, and, since the optic axis is perpendicular to the beam, $n_e = n_\parallel = 1.5534$. The corresponding wavelengths in quartz are $\lambda_o = \lambda_0/n_o = 589.3\ \mathrm{nm}/1.5443 = 381.6$ nm and $\lambda_e = \lambda_0/n_e = 379.4$ nm. The frequencies both equal the vacuum frequency $\nu = c/\lambda_0 = 508.7$ THz.

Hecht 8.39 The Wollaston prism does not rely on total internal reflection, as with the Glan prism pair discussed in class. Rather, the two halves of the prism are cut with differing orientations for the optic axis, so that each polarization component encounters a chnge of index at the interface. The component polarized parallel to the page is the ordinary ray in the first half of the prism, and becomes the extraordinary ray when it crosses the interface into the second half. The corresponding refractive index increase (see the entry for quartz in Table 8.1), and the ray bends toward the normal. The other component experiences a drop in refractive index and bends away from the normal. Note that the diagram differs from that for the calcite Wollaston prism shown in Fig. 8.28.

The angle at which the two prisms are cut, along with the refractive indices, determines the separation between the outgoing beams. Although the displacement of the outgoing beam may be a disadvantage compared to the Glan polarizer, the Wollaston prism is much less sensitive to alignment.



Hecht 8.42 The quarter wave plate is made from a transparent material and will not significantly reduce the intensity of the transmitted light, regardless of polarization. (Also, natural light will remain randomly polarized after transmission.) The polarizer absorbs one of the two polarization components, so transmission through the linear polarizer will reduce the intensity of randomly polarized light by a factor 2.