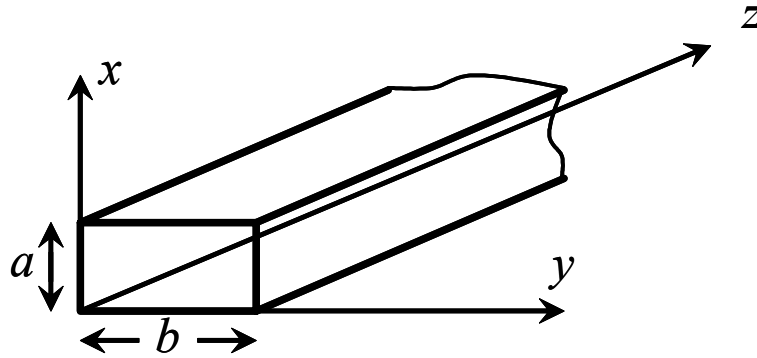


Assignment 6

Physics U603 Summer 1 2005

Due Tuesday, June 13

For the TM modes of a rectangular metal waveguide, the magnetic field is transverse to the propagation direction z , so that $B_z = 0$, but the z -component E_z of the electric field is nonzero.



1. Show that the expression

$$E_z = \frac{\sqrt{k_x^2 + k_y^2}}{k_0} E_0 \sin k_x x \sin k_y y e^{i(k_z z - \omega t)}$$

solves the wave equation

$$\nabla^2 E_z = \frac{1}{c^2} \frac{\partial^2 E_z}{\partial t^2}$$

if

$$k_x^2 + k_y^2 + k_z^2 = k_0^2 = (\omega/c)^2.$$

2. Confirm that this solution satisfies the boundary conditions $E_z(x=0) = E_z(x=a) = 0$ and $E_z(y=0) = E_z(y=b) = 0$ if

$$k_x = \frac{m\pi}{a} \text{ and } k_y = \frac{n\pi}{b},$$

where m and n are integers.

3. Use the appropriate component of the appropriate Maxwell's Equation to determine an expression for B_x .
4. Identify the lowest frequency TM mode for $b > a$. (Be careful: the answer is different than for the TE modes. Why?) Find an expression for the TM cutoff frequency ν_c , that is, the lowest frequency at which electromagnetic waves can propagate down the wave guide.
5. The following table lists the (inside) dimensions of waveguides used in some standard microwave frequency bands. In each case, determine the range of frequencies (*not* angular frequencies) over which the the waveguide will operate in a single mode (*either* TM or TE), so that all energy entering one end of the waveguide arrives at the other end at the same time.

	a (in)	b (in)	frequency range (GHz)
X band	0.400	0.900	
Q band	0.112	0.224	
W band	0.050	0.100	