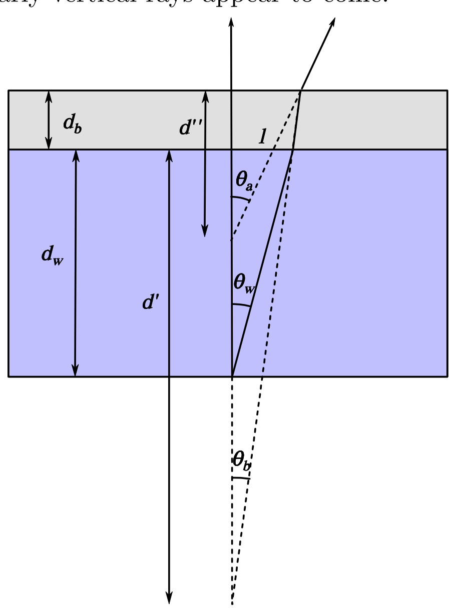
# Model Solution: Homework 4A

Physics U603 Summer 1 2006

**Hecht 4-25:** The pupil of your eye intersects a narrow cone of rays, which extrapolate back to an image position that differs from the actual depth of the coin. You can determine this position by tracing two rays and determining the common point from which they appear to come. Since we are looking from directly above, let's choose one vertical ray and a second ray leaving the coin at a small angle $\theta_w$ from the vertical. Using the small angle aproximation, Snell's Law $n_w\theta_w = n_b\theta_b$ determines the refraction of the latter ray when it reaches the water/benzene interface. In the same approximation, this ray crosses the interface a distance $l = d_w\theta_w$ from the vertical ray that depends on the depth $d_w$ of the water. The extrapolated path of the refracted ray crosses the path of the vertical ray at a distance $d' = l/\theta_b$ below the interface. Substituting for $\theta_b$ (from Snell's law) and for $l$ allows you to determine the depth $d' = (d_w\theta_w) / (n_w\theta_w/n_b) = d_w (n_b/n_w) = 1$ m$(1.5/1.33) = 1.13$ m beneath the interface from which nearly vertical rays appear to come.



These rays refract again when they reach the benzene/air interface. Since these rays are parallel to those coming from an object at a distance $d_b + d'$ below the benzene surface, the same argument can be repeated to determine the apparent depth $d'' = (d_b + d')\,(n_a/n_b) = 1.33\ \text{m}(1/1.5) = 0.89\ \text{m}$.

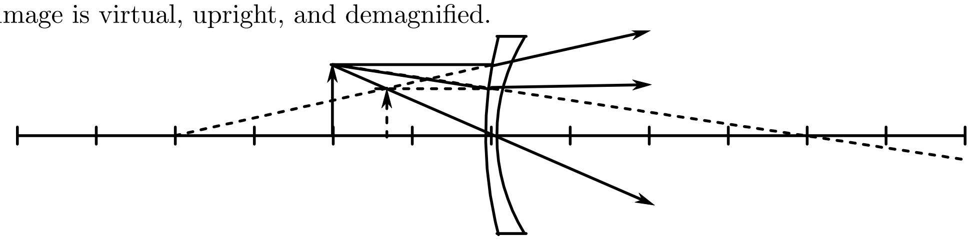
**Hecht 5-12:** With the given values for the index of the glass and the radii of curvature, the lensmaker's formula

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

yields a focal length $f = -40$ cm. The negative sign indicates that this is a diverging lens. Given an object distance $s = 20$ cm, solution of the lens formula

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

yields an image distance $s' = sf/\,(s-f) = (20\ \text{cm})\,(-40\ \text{cm})\,/\,(20+40\ \text{cm}) = -13.3$ cm. The negative sign indicates that the image forms on the same side of the lens as the object, and its magnification is $M_T = -s'/s = +0.667$. The image is virtual, upright, and demagnified.



**Hecht 5-24:**

**(a)** The image distance of the horse's nose is $s_n' = s_n f/\,(s_n - f) = (15\ \text{m})\,(3\ \text{m})\,/\,(15-3\ \text{m}) = 3.75$ m.

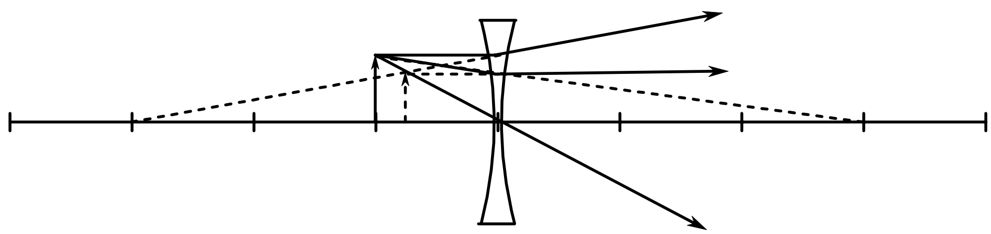
**(b)** The object is *real* because it forms on the opposite side of the lens as the object. Because the refracted rays actually pass through this point, this image can be projected. Since the magnification is $M_T = -s_n'/s_n = -\,(3.75\ \text{m})\,/\,(15\ \text{m}) = -0.25$, the image is *inverted* ($M_T < 0$) and *demagnified* ($|M_T| < 1$).

**(c)** The height $y' = M_T y = (-0.25)\,(2.25\ \text{m}) = -0.5625$ m of the image is reduced by a factor 4 from that of the object.

**(d)** The image of the horse's tail forms a distance $s' = sf/\,(s-f) = (17.5\ \text{m})\,(3\ \text{m})\,/\,(17.5-3\ \text{m}) = 3.62$ m from the lens. As a result, the image measures 3.75 m $-3.62$ m

= 0.13 m from nose to tail. Notice that the longitudinal magnification is very different from the transverse magnification–the nose-to-tail distance has been reduced by a factor 2.5 m/0.13 m = 19, almost five times more than the head-to-foot distance. Fig. 5.26 shows a ray trace of this situation.

**Hecht 5-25:** The image distance is $s' = sf/(s-f) = (10\text{ cm})(-30\text{ cm})/(10+30\text{ cm}) = -7.5$ cm. It is magnified by a factor $-s'/s = -(-7.5\text{ cm})/(10\text{ cm}) = +0.75$, yielding an image height $y' = 0.75y = (0.75)(6.0\text{ cm}) = 4.5$ cm. The image is *virtual*, *upright*, and *demagnified*.



**Hecht 5-58:** Filling a 1.0 cm detector with the image of a 1.0 m object requires a (de)magnification $-s'/s = y'/y = \pm 0.01$. In order to project onto the detector, the image must be real, $s' > 0$, and form at $s' = 0.01s = 0.1$ m. Thus, the detector should be 10 cm from the mirror. The required focal length must satisfy

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'},$$

which yields $f = ss'/(s+s') = (10\text{ m})(10\text{ cm})/(10.1\text{ m}) = 9.9$ cm. It is awkward to draw a ray trace to scale in this case. However, a diagram such as that shown at the top diagram in Fig. 5.53 qualitatively describes the situation–the image of a distant object forms just beyond the focal point of a concave mirror.

**Hecht 5-61:** Since the cornea is a (nearly) spherical surface, light reflected from its surface forms images governed by the usual formulae for image formation by a spherical mirror. A virtual image ($s' < 0$) with magnification $M_T = -s'/s = 0.037$ will form at $s' = -0.037s = -3.7$ mm (behind the the corneal surface). As in the previous problem, this corresponds to a focal length $f = ss'/(s+s') = (100\text{ mm})(-3.7\text{ mm})/(96.3\text{ mm}) = -3.84$ mm, or a radius of curvature $R = 2f = -7.68$ mm.