

Model Solution: Homework 2

Physics U603

Summer 1 2006

1. Hecht 3-7:

- (a) The product of wavelength and frequency gives the phase speed $v_p = \lambda\nu$, so in a vacuum where $v_p = c$, the frequency is $\nu = c/\lambda = (2.998 \times 10^8 \text{ m/s}) / (550 \times 10^{-9} \text{ m}) = 5.45 \times 10^{14} \text{ Hz} = 545 \text{ THz}$.
- (b) $\omega = 2\pi\nu = 3.42 \times 10^{15} \text{ rad/s}$ and $k = 2\pi/\lambda = 1.14 \times 10^7 \text{ rad/m}$.
- (c) The electric and magnetic field amplitudes are related by the wave speed as $B_0 = E_0/c = (600 \text{ V/m}) / (2.998 \times 10^8 \text{ m/s}) = 2.00 \text{ } \mu\text{T}$.
- (d) The general expression for the electric field component of a travelling electromagnetic wave is $\vec{E} = \vec{E}_0 \cos(\vec{k} \cdot \vec{r} - \omega t + \delta)$ or, in complex notation, $\vec{E} = \vec{E}_0 \exp[i(\vec{k} \cdot \vec{r} - \omega t)]$ (with the phase absorbed into the complex amplitude $\vec{E}_0 = |\vec{E}_0| e^{i\delta}$). Since the wave is travelling in the y -direction, $\vec{k} \cdot \vec{r} = ky$, and the electric field amplitude $\vec{E}_0 = \hat{k}E_0$ is parallel to the z -direction. The condition $E(\vec{r} = 0, t = 0) = 0$ restricts the phase to values such that $\cos \delta = 0$, and the natural choice is $\delta = -\pi/2$, so that

$$\vec{E} = \hat{k} (600 \text{ V/m}) \sin [(1.14 \times 10^7 \text{ rad/m}) y - (3.42 \times 10^{15} \text{ rad/s}) t]$$

or

$$\vec{E} = -i\hat{k}E_0 \exp [i (1.14 \times 10^7 \text{ rad/m}) y - (3.42 \times 10^{15} \text{ rad/s}) t].$$

The magnetic field amplitude vector must be directed along x so that $\vec{E} \times \vec{B}$ points along the direction of travel, and

$$\vec{B} = \hat{i} (2.00 \mu\text{T}) \sin [(1.14 \times 10^7 \text{ rad/m}) y - (3.42 \times 10^{15} \text{ rad/s}) t]$$

or

$$\vec{B} = -i\hat{i} (2.00 \mu\text{T}) \exp [i (1.14 \times 10^7 \text{ rad/m}) y - (3.42 \times 10^{15} \text{ rad/s}) t].$$

Note once again the unfortunate notation: the unit vectors \hat{i} and \hat{k} have no connection with the complex unit i or the magnitude of the wavevector \vec{k} .

2. Hecht 3-24: Although the flashlight draws power from the battery at a rate $P = IV = 0.75 \text{ W}$, only 1% of this power, or 7.5 mW, is emitted as light.

- (a) The energy of an individual 550 nm photon is $h\nu = hc/\lambda = 3.61 \times 10^{-19} \text{ J}$. If all emitted photons have this energy, there are $(7.5 \times 10^{-3} \text{ W}) / (3.61 \times 10^{-19} \text{ J}) = 2.08 \times 10^{16} \text{ s}^{-1}$ photons emitted per second.
- (b) Photons within a length L of the beam travel at a speed c will leave this region in a time L/c . Multiplying this time interval by the number of photons emitted per second yields a total number of $(2.08 \times 10^{16} \text{ s}^{-1})(1 \text{ m}) / (2.998 \times 10^8 \text{ m/s}) = 6.93 \times 10^7$ photons in each 1 m section of the beam.
- (c) The flux density, a.k.a. intensity, a.k.a. irradiance, a.k.a. Poynting vector magnitude, is found by dividing the beam power by the cross sectional area of the beam as $I = (7.5 \times 10^{-3} \text{ W}) / (10^{-3} \text{ m}^2) = 7.5 \text{ W/m}^2$.

3. Hecht 3-31:

- (a) The radiation pressure $p_{rad} = 2I/c = 2(1400 \text{ W/m}^2) / (2.998 \times 10^8 \text{ m/s}) = 9.34 \mu\text{N/m}^2$ on a reflecting surface is double what it would be on an absorbing surface.
- (b) If the same reflector is moved to the surface of the Sun, the intensity increase by the square of the distance ratio $[(1.5 \times 10^{11} \text{ m}) / (0.7 \times 10^9 \text{ m})]^2 = 45,900$, and the radiation pressure $p_{rad} = 45,900(9.34 \times 10^{-6} \text{ N/m}^2) = 0.429 \text{ N/m}^2$ increases by the same factor.

4. Hecht 3-38: Rewriting Hecht's expression with brackets and units inserted for clarity,

$$E_z = E_0 \cos \left[\left(\pi \times 10^{15} \text{ rad/s} \right) \left(t - \frac{x}{0.65c} \right) \right].$$

- (a) The frequency is $\nu = \omega/2\pi = 500 \text{ THz}$.
- (b) Since the wave speed is $0.65c$, the wavelength is $\lambda = 0.65c/\nu = 390 \text{ nm}$.
- (c) The index of refraction is the ratio $n = c/v_p = 0.65^{-1} = 1.54$ of the wave speed in the glass to its speed in vacuum.

5. Hecht 3-46: In vacuum, it takes $1.0 \text{ mm}/500 \text{ nm} = 2000$ wavelengths to span a 1 mm distance. In a piece of glass of the same thickness, the wavelength $\lambda = \lambda_0/n$ is reduced from its vacuum value λ_0 by the index of refraction n . The number of wavelengths required to span this distance will increase by the same factor, to 3000 wavelengths for glass with $n = 1.5$. The number of wavelengths spanning the remaining part of the distance

from A to B is unchanged by insertion of the glass. The net effect of the glass is to delay the wave by 1000 cycles, for a phase difference of $2\pi \cdot 1000 = 6280$ rad.