

The Search for Neutrino Masses in Heterotic String Models

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- ⇒ Building a model of flavor from string theory is difficult – neutrinos just a special case
- Assume a particular string construction [heterotic $E_8 \times E_8$]
 - Assume a compactification [symmetric Z_3 orbifold with two Wilson lines]
 - Assume/engineer moduli stabilization at reasonable values

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 - ⇒ Neutrinos give us some hope for a “handle”
 - Neutrino masses may involve a very high energy scale
 - Majorana mass operators are distinct from rest of superpotential
 - Neutrinos may be exotic from the string point-of-view

 - ⇒ Is an existence proof possible?
Can *any* string construction shed light on why neutrinos are so light?

The See-Saw Mechanism(s)

Yanagida (1979); Gell-Mann, Ramond & Slansky (1980)

⇒ See-saw mechanism involves both types of mass terms

$$\mathcal{L} = \frac{1}{2} (\bar{\nu}_L \quad \bar{N}_L^c) \begin{pmatrix} \mathbf{m}_T & \mathbf{m}_D \\ \mathbf{m}_D^T & \mathbf{m}_S \end{pmatrix} \begin{pmatrix} \nu_R^c \\ N_R \end{pmatrix} + \text{h.c.}$$

- Active neutrinos ν_L, N_R (3 flavors each)
- $m_T \Rightarrow$ triplet Majorana mass matrix [SU(2) triplet]
- $m_D \Rightarrow$ Dirac mass matrix [SU(2) doublet]
- $m_S \Rightarrow$ singlet Majorana mass matrix [SU(2) singlet]

⇒ Ordinary (type I) see-saw

- $m_T = 0$ and $m_S \gg m_D$
- $(m_\nu)_{\text{eff}} = -m_D m_S^{-1} m_D^T$ with $U_{\text{PMNS}} = U_e^\dagger U_\nu$
- Ordinary-sterile mixing for m_S and m_D both small and comparable
→ or $m_S \ll m_d$ (pseudo-Dirac)

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- Two discrete Wilson lines a_1 and a_3 , with $a_5 \equiv 0 \Rightarrow$ **three generations**

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- Observable sector gauge group is $G_0 = SU(3) \times SU(2) \times U(1)^5$
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- Identifying embeddings $\{V, a_1, a_3\}$ equivalent under lattice translations and Weyl reflections leave 192 physically distinct cases
- Only 20 different patterns of irreducible representations among these 192 cases

The BSL_A Class: The 20 Subclasses

Pattern	No.	G_H	$r_{FI} = \xi_{FI}/M_{PL}$	δk_Y^{\min}
1.1	7	$SO(10) \times U(1)^3$	No $U(1)_X$	0
1.2	7	$SO(10) \times U(1)^3$	0.216	1/5
2.1	10	$SU(5) \times SU(2) \times U(1)^3$	0.125	4/29
2.2	10	$SU(5) \times SU(2) \times U(1)^3$	0.138	-8/67
2.3	7	$SU(5) \times SU(2) \times U(1)^3$	0.138	0
2.4	7	$SU(5) \times SU(2) \times U(1)^3$	0.186	8/29
2.5	6	$SU(5) \times SU(2) \times U(1)^3$	0.191	11/73
2.6	6	$SU(5) \times SU(2) \times U(1)^3$	0.170	4/11
3.1	12	$SU(4) \times SU(2)^2 \times U(1)^3$	0.148	1/7
3.2	5	$SU(4) \times SU(2)^2 \times U(1)^3$	0.176	-8/119
3.3	10	$SU(4) \times SU(2)^2 \times U(1)^3$	0.170	-4/61
3.4	5	$SU(4) \times SU(2)^2 \times U(1)^3$	0.181	16/59
4.1	7	$SU(3) \times SU(2)^2 \times U(1)^4$	0.138	-8/113
4.2	12	$SU(3) \times SU(2)^2 \times U(1)^4$	0.125	-8/113
4.3	7	$SU(3) \times SU(2)^2 \times U(1)^4$	0.148	8/81
4.4	15	$SU(3) \times SU(2)^2 \times U(1)^4$	0.176	16/61
4.5	17	$SU(3) \times SU(2)^2 \times U(1)^4$	0.157	-1/31
4.6	13	$SU(3) \times SU(2)^2 \times U(1)^4$	0.170	11/73
4.7	6	$SU(3) \times SU(2)^2 \times U(1)^4$	0.157	-1/31
4.8	6	$SU(3) \times SU(2)^2 \times U(1)^4$	0.176	14/5

\Rightarrow Deviation from GUT normalization on hypercharge: $\delta k_Y = k_y - 5/3$

⇒ One of the six cases with representation pattern 2.6

- Embedding $3V = (-1, -1, 0, 0, 0, 2, 0, 0; 2, 1, 1, 0, 0, 0, 0, 0)$
 $3a_1 = (1, 1, -1, -1, -1, 2, 1, 0; -1, 0, 0, 1, 0, 0, 0, 0)$
 $3a_3 = (0, 0, 0, 0, 0, 1, 1, 2; 2, 0, 0, -1, 1, 0, 0, 0)$
- Charge generators for Abelian subgroups $U(1)_a$

a	q_a	$\text{Tr}Q_a$	$k_a/4$
1	$(-3, -3, 2, 2, 2, 0, 0, 0; 0, 0, 0, 0, 0, 0, 0, 0)$	0	15
2	$3(-1, -1, -1, -1, -1, 15, 0, 0; 0, 0, 0, 0, 0, 0, 0, 0)$	0	1035
3	$3(3, 3, 3, 3, 3, 1, -46, 0; 0, 0, 0, 0, 0, 0, 0, 0)$	0	9729
4	$\frac{3}{2}(-3, -3, -3, -3, -3, -1, -1, -47; 0, 0, 0, 0, 0, 0, 0, 0)$	0	2538
5	$\frac{3}{2}(-15, -15, -15, -15, -15, -5, -5, 5; 12, -12, -12, -48, -12, 0, 0, 0)$	0	4590
6	$\frac{1}{2}(-15, -15, -15, -15, -15, -5, -5, 5; -22, -12, -12, 20, 22, 0, 0, 0)$	0	357
7	$3(0, 0, 0, 0, 0, 0, 0, 0; 1, 0, 0, 0, 1, 0, 0, 0)$	0	9
X	$\frac{1}{2}(-3, -3, -3, -3, -3, -1, -1, 1; 4, 6, 6, 4, -4, 0, 0, 0)$	504	21

- Anomalous $U(1)$ FI-term: $\xi_{\text{FI}} = \frac{g_{\text{STR}}^2 \text{Tr}Q_X}{192\pi^2} M_{\text{PL}}^2 = 0.170 M_{\text{PL}}^2$

A Typical Model

⇒ 51 species of chiral superfields

- One quark doublet in untwisted sector
- Three candidate $\bar{3}$ representations of $SU(3)_c$
- Six candidate 2 representations of $SU(2)_L$
- One bifundamental $(2, 2)$ with respect to $SU(2)_L \times SU(2)_H$
- 25 species with only Abelian charges ⇒ *all candidate N_R species!*

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⇒ No unique hypercharge assignment

- No particular linear combination of $U(1)_a$ singled out as $U(1)_Y$
- Always a family of solutions for any particular field assignment
- Rule of thumb: pick an assignment that minimizes δk_Y
- In this case 18 different assignments of $\{Q, u, d, L, e, H_u, H_d\}$ have $\delta k_Y \leq 1$

⇒ Species can be grouped by fixed point location in first two complex planes

- For third complex plane, fixed point location = family index

Sample Spectrum (Part One)

No.	Irrep	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_X	Y
1	$(3, 2, 1, 1)_U$	1	6	-18	9	45	15	0	3	1/6
2	$(1, 2, 1, 1)_U$	3	18	-54	27	-45	-15	0	-3	1/2
3	$(\bar{3}, 1, 1, 1)_U$	-4	-24	72	-36	0	0	0	0	-2/3
4	$(1, 1, 10, 2)_U$	0	0	0	0	-18	-6	0	3	0
5	$(1, 1, 5, 1)_U$	0	0	0	0	36	12	0	-6	0
6	$(1, 1, 1, 1)_{-1,-1}$	0	-20	-32	-31	-35	-23	0	1	0
7	$(1, 1, 1, 1)_{-1,-1}$	0	-35	13	17	25	-3	0	5	2/5
8	$(1, 1, 1, 1)_{-1,-1}$	0	10	16	-55	25	-3	0	5	-2/5
9	$(1, 1, 1, 1)_{-1,-1}$	0	10	-122	14	10	-8	0	4	0
10	$(\bar{3}, 1, 1, 1)_{-1,-1}$	2	7	25	11	-5	-13	0	3	1/3
11	$(1, 2, 1, 1)_{-1,-1}$	-3	7	25	11	-5	-13	0	3	-1/2
12	$(1, 1, 1, 2)_{-1,0}$	0	-5	61	-7	-5	-13	0	-4	0
13	$(1, 1, 1, 1)_{-1,0}$	0	-5	61	-7	-95	-9	0	1	0
14	$(1, 1, 1, 2)_{-1,0}$	0	-20	-32	-31	55	7	0	0	0
15	$(1, 1, 1, 1)_{-1,0}$	0	-20	-32	-31	-35	11	0	5	0
16	$(1, 1, 1, 2)_{-1,0}$	0	25	-29	38	40	2	0	-1	0
17	$(1, 1, 1, 1)_{-1,0}$	0	25	-29	38	-50	6	0	4	0
18	$(1, 1, 1, 2)_{-1,1}$	0	-5	61	-7	-5	21	0	0	0
19	$(1, 1, \bar{5}, 1)_{-1,1}$	0	-5	61	-7	49	5	0	1	0
20	$(1, 1, 1, 1)_{-1,1}$	0	-5	61	-7	-5	4	-3	5	-1/5
21	$(1, 1, 1, 1)_{-1,1}$	0	-5	61	-7	-5	4	3	5	1/5
22	$(1, 1, 1, 1)_{0,-1}$	2	-28	38	-19	-5	4	-1	5	2/5
23	$(1, 1, 1, 1)_{0,-1}$	2	17	41	50	-20	-1	-1	4	2/5
24	$(1, 1, 1, 1)_{0,-1}$	2	17	-97	-22	-20	-1	-1	4	0
25	$(1, 2, 1, 1)_{0,-1}$	-1	14	50	-25	-35	-6	-1	3	-1/2
26	$(1, 1, 1, 1)_{0,-1}$	-4	-4	-34	17	-5	4	-1	5	-3/5

Sample Spectrum (Part Two)

No.	Irrep	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_X	Y
27	$(3, 1, 1, 1)_{0,-1}$	0	-10	-16	8	-50	-11	-1	2	1/15
28	$(1, 1, 1, 2)_{0,0}$	2	32	-4	2	10	9	-1	-1	0
29	$(1, 1, 1, 1)_{0,0}$	2	32	-4	2	10	-8	2	4	1/5
30	$(1, 2, 1, 2)_{0,0}$	-1	-16	2	-1	-5	4	-1	-2	-1/10
31	$(1, 2, 1, 1)_{0,0}$	-1	-16	2	-1	-5	-13	2	3	1/10
32	$(1, 1, \bar{5}, 1)_{0,1}$	2	2	-52	26	4	7	-1	0	2/5
33	$(1, 1, 1, 2)_{0,1}$	2	2	-52	26	40	2	2	-1	3/5
34	$(1, 1, 1, 1)_{0,1}$	2	2	-52	26	40	-15	-1	4	2/5
35	$(1, 1, 1, 1)_{0,1}$	2	2	-52	26	-50	6	2	4	3/5
36	$(1, 1, 1, 2)_{1,-1}$	-2	3	-9	-19	55	7	-2	0	-3/5
37	$(1, 1, \bar{5}, 1)_{1,-1}$	-2	3	-9	-19	19	12	1	1	-2/5
38	$(1, 1, 1, 1)_{1,-1}$	-2	3	-9	-19	-35	11	-2	5	-3/5
39	$(1, 1, 1, 1)_{1,-1}$	-2	3	-9	-19	55	-10	1	5	-2/5
40	$(1, 1, 1, 1)_{1,0}$	-2	-27	-57	5	-5	4	1	5	0
41	$(1, 1, 1, 1)_{1,0}$	-2	18	84	5	-5	4	1	5	-2/5
42	$(\bar{3}, 1, 1, 1)_{1,0}$	0	15	-45	-1	-35	-6	1	3	-1/15
43	$(1, 1, 1, 1)_{1,0}$	4	-6	18	38	-20	-1	1	4	1
44	$(1, 2, 1, 1)_{1,0}$	1	-9	27	-37	-35	-6	1	3	1/10
45	$(1, 1, 1, 1)_{1,0}$	-2	-12	36	29	-65	-16	1	1	0
46	$(1, 1, 1, 2)_{1,1}$	-2	-12	36	29	25	14	1	0	0
47	$(1, 1, 1, 1)_{1,1}$	-2	-12	36	29	25	-3	-2	5	-1/5
48	$(1, 1, 1, 2)_{1,1}$	4	9	-27	-10	10	9	1	-1	3/5
49	$(1, 1, 1, 1)_{1,1}$	4	9	-27	-10	10	-8	-2	4	2/5
50	$(1, 1, 1, 2)_{1,1}$	-2	3	-9	-19	-35	-6	1	-4	-2/5
51	$(1, 1, 1, 1)_{1,1}$	-2	3	-9	-19	-35	-23	-2	1	-3/5

- ⇒ Not sufficient to merely compute the spectrum – we need the superpotential
- Superpotential couplings determined by string *selection rules*; examples:
 - ★ Point group selection rule [twisted states come in groups of 3]
 - ★ Lattice group selection rule [fixed point locations must sum to 0 mod 3]
 - Not all Dirac-type couplings will arise at the leading (trilinear) order

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- Not all Dirac-type couplings will arise at the leading (trilinear) order
- No fundamental (SUSY) mass terms

String constructions may be able to generate large effective m_S

$$W_\nu \sim c_{ij} \frac{S^{q+1}}{M_{Pl}^q} N_i N_j \quad \Rightarrow \quad (m_S)_{ij} \sim c_{ij} \frac{\langle S \rangle^{q+1}}{M_{Pl}^q}$$

⇒ Effective couplings we seek will involve vacuum expectation values

⇒ These vevs must lie along flat directions to preserve SUSY

⇒ **Step 1:** Classify all D-flat ($\langle D_a \rangle = 0$) directions with certain properties

- Each holomorphic gauge invariant $I(\Phi_1, \dots, \Phi_n)$ defines a family of conditions

$$\langle K_i \rangle = c \langle I_i \rangle \text{ with } K_i = \partial K / \partial \Phi_i \text{ and } I_i = \partial I / \partial \Phi_i$$

- The invariant I is required to have $Q_X < 0$ and be neutral under $SU(3) \times SU(2) \in G_O$

$$D_X = \sum_i K_i q_i^X \phi^i + \xi_{\text{FI}} \text{ with } \xi_{\text{FI}} = \frac{g_{\text{STR}}^2 \text{Tr} Q_X}{192\pi^2} M_{\text{PL}}^2$$

- We build all such invariants with degree less than or equal to ten
- Did not consider generalizations which may involve polynomials (and allow more fields to obtain vevs):

$$\langle K_i \rangle = \sum_{\alpha} c_{\alpha} \langle I_i^{\alpha} \rangle$$

Classifying Flat Directions

⇒ **Step 2:** Generate all superpotential couplings allowed by selection rules up to and including degree 9

Pattern	3	4	6	7	8	9
1.1	113	24	21329	23768	1697	3380308
1.2	97	12	13968	4418	498	1552812
2.1	67	10	5188	3515	162	342186
2.2	80	11	7573	3066	272	582326
2.3	75	10	6508	2874	250	467020
2.4	53	0	2795	360	0	119454
2.5	58	6	3363	688	26	150838
2.6	31	0	642	0	0	10976
3.1	54	4	2749	768	21	119973
3.2	43	2	1758	291	9	59182
3.3	48	4	2187	393	20	81497
3.4	31	8	750	375	42	15074
4.1	50	3	2090	693	14	81222
4.2	62	6	3206	793	38	143257
4.3	55	5	2516	613	15	100793
4.4	38	2	1137	147	3	28788
4.5	48	0	1872	0	0	62597
4.6	47	0	1738	50	0	51970
4.7	53	0	2219	0	0	76244
4.8	21	0	301	0	0	4120

⇒ Many of the higher order couplings just products of lower order invariants

⇒ **Step 3:** Eliminate from the list of I-monomials all those that would violate F-flatness with respect to couplings generated in Step 2

- To check F-flatness: if $W = \sum_{\alpha} \lambda_{\alpha} W^{\alpha}$ we demand

$$\left\langle \frac{\partial W^{\alpha}}{\partial \phi_i} \right\rangle = 0 ; \forall \alpha, i$$

- This is not allowing for cancellations among terms in the superpotential

$$\langle F_i \rangle = \left\langle \sum_{\alpha} \lambda_{\alpha} \frac{\partial W^{\alpha}}{\partial \phi_i} \right\rangle = 0 ; \forall i$$

- ★ Technical issue: λ_{α} 's
- ★ Naturalness issue: $\langle \phi_i \rangle$'s

Pattern	w/o	w/ 3	w/ 3-9
1.1	1486616	16283	489
1.2	11656	188	28
2.1	155555	1239	245
2.2	96932	737	249
2.3	43884	670	115
2.4	5195	114	12
2.5	12	0	0
2.6	825	9	9
3.1	16927	80	27
3.2	2443	18	10
3.3	9871	74	22
3.4	1303	59	41
4.1	17413	106	26
4.2	78819	513	199
4.3	14715	310	163
4.4	26	0	0
4.5	5126	32	25
4.6	128	8	5
4.7	5285	15	15
4.8	49	1	1

- For each surviving flat direction, want to find terms of form

$$W \ni \langle S_1 \cdots S_n \rangle NN$$

where vevs $\langle S_i \rangle$ reside in an invariant I for a given flat direction

\Rightarrow *This happened only twice: patterns 1.1 and 2.6*

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- Are Dirac couplings to doublets present along the flat direction as well?

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 1. Can we assign hypercharge correctly to L , H_u and N_R ?
 2. While keeping the flat direction from breaking this $U(1)$?

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- But that is still not enough...
 1. Can we assign hypercharge correctly to L , H_u and N_R ?
 2. While keeping the flat direction from breaking this $U(1)$?
 3. Can we add in the charged lepton and H_d doublet?
 4. Can we add in the quark sector?
 5. And do any of these SM fields get projected out along the flat direction?

- Pattern 2.6: candidate Majorana couplings at degree **3** and **9**

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 - ★ 14 total Majorana candidates along 6 of the 9 flat directions
 - ★ All six allow $W \ni \phi_4 \phi_5 \phi_5 \Rightarrow \mathbf{5} \mathbf{10} \mathbf{10}$ of $SU(5)_H$
 - ★ All others involve $\langle \phi_{43}^3 \rangle$ and “neutrinos” are either ϕ_{24} or ϕ_{34}

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 - ★ All others involve $\langle \phi_{43}^3 \rangle$ and “neutrinos” are either ϕ_{24} or ϕ_{34}
...all are $\bar{\mathbf{5}}$'s of $SU(5)_H$!

No.	Irrep	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_X
S_4	$(1, 1, \mathbf{5}, 1)_{\text{untw}}$	0	0	0	0	72	0	72	-36
N_5	$(1, 1, \mathbf{10}, 2)_{\text{untw}}$	0	0	0	0	-36	0	-36	18
L_{12}	$(1, 2, 1, 1)_{-1,0}$	-30	2	0	0	-70	-2	-36	18
N_{24}	$(1, 1, \bar{\mathbf{5}}, 1)_{0,-1}$	-12	14	2	0	98	0	30	6
B_{30}	$(1, 2, 1, 2)_{0,0}$	6	26	0	0	-10	-2	24	-12
N_{34}	$(1, 1, \bar{\mathbf{5}}, 1)_{0,1}$	-12	14	-2	0	38	2	72	6
S_{43}	$(1, 1, \bar{\mathbf{5}}, 1)_{1,0}$	24	-28	0	0	8	-2	42	0
L'_{44}	$(1, 1, 1, 2)_{1,0}$	24	-28	0	0	80	4	12	-6

- Example of a “Dirac” coupling: $\langle S_4 \rangle N_5 N_5 L_{12} B_{30} L'_{44}$ (3 of 642 at $d = 6$)
- Fields N_{24} and N_{34} have no couplings to doublets through degree 6

- Pattern 2.6: candidate Majorana couplings at degree 3 and 9
 - ★ 14 total Majorana candidates along 6 of the 9 flat directions
 - ★ All six allow $W \ni \phi_4 \phi_5 \phi_5 \Rightarrow \mathbf{5} \mathbf{10} \mathbf{10}$ of $SU(5)_H$
 - ★ All others involve $\langle \phi_{43}^3 \rangle$ and “neutrinos” are either ϕ_{24} or ϕ_{34}
...all are $\bar{\mathbf{5}}$'s of $SU(5)_H$!

No.	Irrep	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_X
S_4	$(1, 1, \mathbf{5}, 1)_{\text{untw}}$	0	0	0	0	72	0	72	-36
N_5	$(1, 1, \mathbf{10}, 2)_{\text{untw}}$	0	0	0	0	-36	0	-36	18
L_{12}	$(1, 2, 1, 1)_{-1,0}$	-30	2	0	0	-70	-2	-36	18
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- Example of a “Dirac” coupling: $\langle S_4 \rangle N_5 N_5 L_{12} B_{30} L'_{44}$ (**3** of 642 at $d = 6$)
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$$m_\nu \sim \frac{(r_{\text{FI}}^6 v_u)^2}{r_{\text{FI}}^7 M_{\text{PL}}} \sim r_{\text{FI}}^5 \times 10^{-5} \text{eV} \Rightarrow \text{too small! } (r_{\text{FI}} \sim 0.17)$$

- Pattern 1.2: candidate Majorana couplings at degree 6 and 8
- Embedding $3V = (-1, -1, 0, 0, 0, 2, 0, 0; 2, 1, 1, 0, 0, 0, 0, 0)$
 $3a_1 = (1, 1, -1, -1, 2, 0, 0, 0; 0, 2, 0, 0, 0, 0, 0, 0)$
 $3a_3 = (0, 0, 0, 0, 0, 0, 2, 0; -1, 0, -1, 0, 0, 0, 0, 0)$
- ★ 42 total Majorana couplings along 18 distinct flat directions
- ★ These couplings involve 8 different candidate N_R species

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 - ★ 42 total Majorana couplings along 18 distinct flat directions
 - ★ These couplings involve 8 different candidate N_R species
- ⇒ 3 candidate N_R have potential Dirac couplings at leading order (degree 3), three more at degree 4
- An example flat direction

I-monomial:	$(3, 3, 8, 21, 22, 29, 46, 72)$
Eff. Maj. mass (1):	$\langle S_8 S_{22} S_{46} S_{72} \rangle N_9 N_9$
Eff. Maj. mass (2):	$\langle S_3 S_3 S_8 S_{22} S_{46} S_{72} \rangle N_{13} N_{13}$
- Three of the eight $U(1)$ factors remain unbroken along this direction

Candidate Yukawa Couplings

No.	Irrep	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
S_3	$(1, 1, 1)_{\text{untw}}$	0	0	-6	0	6	0	0	0
L_5	$(1, 2, 1)_{-1,-1}$	18	4	-2	2	0	-2	-2	0
N_9	$(1, 1, 1)_{-1,-1}$	0	-5	-5	-1	3	-2	-2	0
N_{13}	$(1, 1, 1)_{-1,-1}$	0	-5	1	-1	-3	-2	-2	0
L_{36}	$(1, 2, 1)_{0,-1}$	-6	2	4	2	0	-2	2	0
S_{38}	$(1, 1, 1)_{0,-1}$	-24	-7	1	-1	3	-2	2	0
L_{52}	$(1, 2, 1)_{0,1}$	-6	2	4	-2	0	0	2	2
S_{60}	$(1, 1, 1)_{1,-1}$	-12	-6	-2	-4	0	4	0	0
L_{64}	$(1, 2, 1)_{1,-1}$	6	3	1	-1	-3	4	0	0
L_{71}	$(1, 2, 1)_{1,0}$	6	3	1	3	-3	2	0	-2

- Two (equivalent) pairs of Dirac mass couplings possible

$$(A) \begin{cases} N_9 L_{36} L_{64} \\ \underline{S_3} N_{13} L_{36} L_{64} \end{cases} \quad (B) \begin{cases} N_9 L_{52} L_{71} \\ \underline{S_3} N_{13} L_{52} L_{71} \end{cases}$$

No.	Irrep	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
S_3	$(1, 1, 1)_{\text{untw}}$	0	0	-6	0	6	0	0	0
L_5	$(1, 2, 1)_{-1, -1}$	18	4	-2	2	0	-2	-2	0
N_9	$(1, 1, 1)_{-1, -1}$	0	-5	-5	-1	3	-2	-2	0
N_{13}	$(1, 1, 1)_{-1, -1}$	0	-5	1	-1	-3	-2	-2	0
L_{36}	$(1, 2, 1)_{0, -1}$	-6	2	4	2	0	-2	2	0
S_{38}	$(1, 1, 1)_{0, -1}$	-24	-7	1	-1	3	-2	2	0
L_{52}	$(1, 2, 1)_{0, 1}$	-6	2	4	-2	0	0	2	2
S_{60}	$(1, 1, 1)_{1, -1}$	-12	-6	-2	-4	0	4	0	0
L_{64}	$(1, 2, 1)_{1, -1}$	6	3	1	-1	-3	4	0	0
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- Both combinations allow for charged lepton and Higgs bilinear couplings

$$\text{e.g. } W \ni \lambda_\nu N L_{36} L_{64} + \lambda_e L_{36} L_5 S_{60} + \lambda_S L_{64} L_5 S_{38}$$

$$\text{or } W \ni \lambda_\nu N L H_u + \lambda_e L H_d E^c + \lambda_S S H_u H_d$$

where we identify $\{L_{36}, L_{64}, L_5, S_{60}, S_{38}\} \leftrightarrow \{L, H_u, H_d, E^c, S\}$

- A (partially) successful hypercharge assignment can be found

$$U(1)_Y = -\frac{7}{180}U(1)_1 - \frac{1}{30}U(1)_2 + \frac{1}{6}U(1)_4 + \frac{1}{4}U(1)_6 - \frac{1}{4}U(1)_7 + \frac{1}{12}U(1)_8$$

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- ★ All $U(1)$ combinations with $k_Y = 5/3$ broken by this flat direction...

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- ★ Quark doublets Q and triplets U and D can also be accommodated
 - ★ But hypercharge normalization is way off: $k_Y = 91/6$
 - ★ All $U(1)$ combinations with $k_Y = 5/3$ broken by this flat direction...
- Quark masses problematic as well
 - ★ Q_1 in untwisted sector, but $\{L_{36}, L_{64}, L_{52}, L_{71}\}$ in twisted sector
 - ⇒ No leading order Yukawas for quark masses
 - ★ First coupling involving Q_1 : $W^6 \ni \langle S_3 S_8 S_{72} \rangle Q_1 Q_1 D_{56}$

No.	Irrep	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
Q_1	$(3, 2, 1)_{\text{untw}}$	6	-12	0	0	0	0	0	0
D_{56}	$(3, 1, 1)_{0,1}$	0	5	1	1	-3	0	2	2

- Flat direction projects out many $SU(2)$ doublets (and $SU(3)$ triplets)

$$W = \lambda_1 \underline{S_{21}} L_{49} L_{70} + \lambda_2 \underline{S_{22}} L_{12} L_{24} + \lambda_3 \underline{S_{29}} L_{51} L_{80} + \lambda_4 \underline{S_{46}} L_{47} L_{48}$$

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- But no distinction between $N_9 \leftrightarrow N_{13}$ and $\{L_{36}, L_{64}\} \leftrightarrow \{L_{52}, L_{71}\}$
- Candidate N with Dirac couplings mix with states \tilde{N} that do not

$$\begin{aligned} W_{\text{mix}} = & \lambda \underline{S}_8 N_9 \tilde{N}_{14} + \lambda \underline{S}_3 \underline{S}_8 N_{13} \tilde{N}_{14} + \lambda \underline{S}_{22} N_9 \tilde{N}_{27} + \lambda \underline{S}_3 \underline{S}_{22} N_{13} \tilde{N}_{27} \\ & + \lambda \underline{S}_{72} N_9 \tilde{N}_{50} + \lambda \underline{S}_3 \underline{S}_{72} N_{13} \tilde{N}_{50} + \lambda \underline{S}_{46} N_9 \tilde{N}_{81} + \lambda \underline{S}_3 \underline{S}_{46} N_{13} \tilde{N}_{81} \end{aligned}$$

\Rightarrow *Neutral lepton sector larger than the minimal see-saw!*

$$\begin{aligned} \nu_L &= \{(\nu_L)_{36}, (\nu_L)_{52}\}, \\ \tilde{N} &= \{\tilde{N}_{14}, \tilde{N}_{27}, \tilde{N}_{50}, \tilde{N}_{81}\}, \\ N &= \{N_9, N_{13}\} \end{aligned}$$

- No Dirac couplings of \tilde{N} states to lepton doublets to degree 6
- No Majorana couplings of \tilde{N} states to themselves to degree 9

⇒ Effective neutrino mass matrix to degree 6:

$$(\nu_L \quad \tilde{N} \quad N) \begin{pmatrix} 0 & 0 & A \\ 0 & 0 & B \\ A & B & C \end{pmatrix} \begin{pmatrix} \nu_L \\ \tilde{N} \\ N \end{pmatrix}$$

⇒ Mass matrices determined by vevs with $A \ll C \ll B$

$$A = \begin{pmatrix} \langle (H_u)_{64} \rangle & \langle S_3(H_u)_{64} \rangle \\ \langle (H_u)_{71} \rangle & \langle S_3(H_u)_{71} \rangle \end{pmatrix}$$

$$B = \begin{pmatrix} \langle S_8 \rangle & \langle S_3 S_8 \rangle \\ \langle S_{22} \rangle & \langle S_3 S_{22} \rangle \\ \langle S_{72} \rangle & \langle S_3 S_{72} \rangle \\ \langle S_{46} \rangle & \langle S_3 S_{46} \rangle \end{pmatrix}$$

$$C = \begin{pmatrix} \langle S_8 S_{22} S_{46} S_{72} \rangle & 0 \\ 0 & \langle S_3^2 S_8 S_{22} S_{46} S_{72} \rangle \end{pmatrix}$$

⇒ First $m_{AB} \tilde{N}_A \tilde{N}_B$ $A \neq B$ term arises at degree 12 ⇒ $m_\nu \sim 10^{-13} \text{eV}$

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- Faraggi et al. [PLB 245 \(1990\) 435](#); [PLB 307 \(1993\) 311](#); [PLB 581 \(2004\) 99](#)
 - ★ Extended neutrino sector (seven additional N' fields)
 $W \sim \langle S_1 \cdots S_{n-2} \rangle NN'$
 - ★ NN' coupling determined by hidden sector strong dynamics (flat direction unclear)
 - ★ See-saw fulcrum requires breaking some additional $U(1)'$ at low scale

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 - ★ Flat direction chosen forbids Dirac couplings for one generation of neutrino multiplet
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 - ★ Flat direction chosen forbids Dirac couplings for one generation of neutrino multiplet
 - ★ For this generation must utilize neutrino/singlet mixing
 - ★ Majorana masses require strong dynamics in hidden sector
- Our approach a slightly different one
 - ★ These studies pick a flat direction and do what they can...
 - ★ We search all flat directions looking only for the minimal see-saw

- Where to turn next?
 - ★ More general flat directions?
 - ★ Even higher order? Other constructions?
 - ★ Extended see-saws?
 - ★ Very, very high order Dirac masses?
 - ★ Triplet case: jettison BSL_A for $SU(3) \times SU(2)^2$ observable sector

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- What have we learned by these exercises?
 - ★ Flat direction analysis *crucial* to meaningful statements
 - ★ Interesting redundancies – vacuum not as vast as we thought!
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 - ★ Small neutrino masses certainly not an *automatic* feature of string constructions
 - ★ Even promising constructions can fail when the *full* system of relevant states is identified
- Nevertheless, meaningful statements about string models are possible *provided* we go in armed with *concrete* questions!